C: Answers to Selected Problems

Chapter 1.1, Propositions and Truth Tables

1. [a], [b], [d], [e].

3. None of the sentences can be assigned a truth value because [a] is a command, [b] is a command, [c] is a question, and [d] contains an unknown variable.

5. [a] Inga has two aces in her card hand and she has a full house.
   [c] Inga does not have a full house.

7. Because the president could be from a third party.

9. 16 combinations.

11. [a] is a true statement, [b] is not a statement, [c] is not a statement, [d] is a false statement.

13. [a] is False, [b] is False, [c] is False, [d] is False, [e] is False, [f] is False.

15. [a] is True, [b] is False, [c] is False, [d] is False, [e] is True, [f] is True.

17. The compound statements are equivalent (produce the same truth values under identical conditions).

19. [a] $p \lor q$, [b] $p \land \neg q$, [c] $(p \lor q)$, [d] $\neg (p \land q)$.

21. [a]  

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[b] and [c] are not logically equivalent.

23. Yes.


27. There are many possible answers, one is $\neg (p \land q)$.

29. The statements are not logically equivalent.

31. Person 2 speaks the truth and Person 1 and Person 3 are liars.
Chapter 1.2, Conditional Statements and Arguments

1. [a] \( p \lor q \rightarrow r \), [b] \( p \leftrightarrow s \), [c] \( \neg q \rightarrow \neg r \), [d] \( p \land \neg r \).

3. [a] \[
\begin{array}{|c|c|c|c|}
\hline
p & q & r & (p \lor q) \rightarrow r \\
\hline
T & T & T & T \\
T & T & F & F \\
T & F & T & T \\
T & F & F & F \\
F & T & T & T \\
F & T & F & F \\
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F & F & F & T \\
\hline
\end{array}
\]

[c] \[
\begin{array}{|c|c|c|}
\hline
q & r & \neg q \rightarrow \neg r \\
\hline
T & T & T \\
T & F & T \\
F & T & F \\
F & F & T \\
\hline
\end{array}
\]

5. \( p \rightarrow q \) and \( \neg q \rightarrow \neg p \) are logically equivalent, so are \( q \rightarrow p \) and \( \neg p \rightarrow \neg q \).

7. [a] If \( 2 + 3 \) does not equal 10, then Indianapolis is the capital of Indiana.
   [b] If \( 2 + 3 = 10 \), then Indianapolis is not the capital of Indiana.
   [c] If Indianapolis is the capital of Indiana, then \( 2 + 3 \) does not equal 10.


13. Two statements are logically equivalent.

15. Not a valid argument.

17. [a] If I cannot play hockey, then I did not finish my homework.
   [b] If I play hockey, then I finished my homework.
   [c] If I do not finish my homework, then I cannot play hockey.

19. Not logically equivalent. Placement of the parentheses is important.


23. Valid argument.

25. Statement is a tautology.

27. Not a valid argument.

29. Not a valid argument.

31. [a] It snowed and practice was not cancelled.
   [b] We swim if and only if the lifeguard is not present. or
      We don't swim if and only if the lifeguard is present.
Chapter 1.3, Quantifiers

1. [a] \( P = \{2, 3, 5, 7\} \), [b] \( Q = \{1, 2, 3\} \), [c] \( S = \{2, 4, 6, 8\} \), [d] \( T + \{1, 2, 3, 4, 5, 6\} \).
2. [a] \( (\forall x) (p(x)); \) False, [b] \( (\exists x) (q(x)); \) True, [c] \( (\exists x) (\neg p(x)); \) True,
   [d] \( (\exists x) (p(x) \land \neg r(x)); \) True, [e] \( (\forall x) (p(x) \land q(x)); \) False, [f] \( (\exists x) (p(x) \land q(x)); \) True.
4. No.
5. Yes, \( P = \{x \mid x \text{ is a carnivore}\} \).

Chapter 1.4, Chapter Review

Mastery Quiz

1. [a], 2. [b], 3. [c], 4. [d], 5. [b], 6. [c], 7. [a], 8. [a], 9. [b], 10. [a]

Review

1. Both signatures are required.
2. [a] \( \neg p \land q \land r \), [c] \( q \leftrightarrow p \).
3. \( [a] \text{ is true}, [c] \text{ is true.} \)
4. \( [a] \text{ and } [b] \text{ are true.} \)
5. \( [a], [c], \text{ and } [d] \text{ are tautologies.} \)
6. \( [a], [b], \text{ and } [c] \text{ are tautologies.} \)
7. Yes.
8. Is not a valid argument.
9. \( \neg p \leftrightarrow \neg q \).
10. Is not a valid argument.
11. If it pours, it rains.
12. Is a valid argument.
13. Is not a valid argument.
15. Yes.
16. Is not a valid argument.
Chapter 2.1, Sets and Subsets


3. This is not a well-defined set.

5. All students with blue eyes. All students who are male. All students who own a dog.

7. \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}.


11. \{x \mid \text{you add the two adjacent elements to get the next element between 0 and 100}\}

13. 7.


17. Yes, the empty set is always a subset of any set.

19. [b] and [c] are well-defined sets.

21. B = \{6, 8, 10, 12\}.


25. W = \{1, 2, 3, 4\}.

27. There are 29 different coin combinations.

29. 4 possible subsets: \{\}, \{a\}, \{b\}, \{a, b\}.

31. There are 16 possible subsets.

33. \{1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, 24, 25, 30, 36\}.

35. \{0, 1, 2, 3, 4, 5, 6\}.

37. 60 wolves are on the island.

39. \{(Y, Y, Y, N), (Y, Y, N, Y), (Y, N, Y, Y), (N, Y, Y, Y), (Y, Y, Y, Y)\}.

Chapter 2.2, Set Operations

1. \(A \cup B = \{a, b, c, e, g, n, t, w\}\), \(A \cap B = \{b, e, f\}\).

3. \(C = \{\text{Jose, Beverly, Tom, Phil, Sue}\}\).

5. \( \{\}\), The empty set.

7. \(M - N = \{\text{Mike, Jody}\}\).

9. \(P - Q = \{3, 7\}\), \(Q - P = \{9\}\).

11. [a] \((H \cup K)' = \{c, d, e, o\}\), [b] \(H' \cap K' = \{c, d, e, o\}\), [c] \((H \cap K)' = \{b, c, d, e, o, u\}\), [d] \(H' \cup K' = \{b, c, d, e, o, u\}\).


15. \{Nick, Linda, Mike, Jody\}.

17. [a] \((A')' = \{1, 2, 3, 4\} = A\), [b] \((A \cap B) \cup (B \cap C) = \{3, 4, 5, 6\} = B\)

[c] \(A \cup B \cap C = \{5, 6\}\).
19. \( W = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\} \)
21. \([a] \{c, e, f\}, [b] \{e\}, [c] \{(a, a), (a, f), (c, a), (c, f)\}\)
23. \( S = \{(H, H), (H, T), (T, H), (T, T)\}\)
25. \( A = \{1, 3, 5, 7, 9\}, B = \{2, 3, 5, 7\}, C = \{6, 7, 8, 9\}. \)
27. 17.
29. \( S = \{HHH, HHT, HTT, THH, TTH, THT, TTT\}. \)

Chapter 2.3, Venn Diagrams

1. \(\{x, y, z\}\)
3. \((A \cup B)'\) or \(A' \cap B'\).
5. \(\{w\}\)
7. 2.
9. No.
11. The region shaded is outside both subsets.
13. You cannot find the complement to any set without knowing the elements in the universal set.
15. 19.
17. 50.
19. \((12 + 18) - 25 = 5\).
21. \(72 - (37 + 32 - 9) = 12\).
23. 20.
25. \(45 = n(A) + 2 \cdot 2n(A) + 3 \cdot 2 \cdot 2n(A), n(A) = 5\).
27. 15.
29. \([a] 34, [b] 15, [c] 24\).
31. 12.
33. \(4 \cdot 5 \cdot 3 = 60\).
35. \(A = B = C\).

Chapter 2.4, Chapter Review

Mastery Quiz

1. \([a, d], 2. [b, c, d], 3. [b], 4. [d], 5. [d], 6. [d], 7. [c], 8. [d], 9. [a], 10. [a]\)

Review

1. \(\{(c, a), (c, e), (c, u), (u, a), (u, e), (u, u)\}\).
3. \([a] \{4, 6\}, \[c] \{(2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6), (5, 4), (5, 7), (7, 4), (7, 5)\}, \[7, 6\}, \[c] \{3, 5, 7\}\).
5. \(2^k\), where \(k\) is the cardinality.
7. 48.
9. \(\{1, 2\}\).
11. \(39 + 43 - 27 = 55\).
13. 19.
15. The element \((a, a)\) is the only one.
17. 22.
19. \([a] 40, [b] 10\).
21. \(A = \{3, 4, 5, 8, 9, 11\}, B = \{2, 5, 6, 8, 10\}, C = \{3, 6, 7, 8, 11\}\).
Chapter 3.1, Trees and Equally Likely Outcomes
1. \( S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\} \).
2. \( S = \{1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, 24, 25, 30, 36\} \). Remember, repeated elements are rostered only once in a set.
3. \( 6 \cdot 2 = 12, S = \{1H, 1T, 2H, 2T, 3H, 3T, 4H, 4T, 5H, 5T, 6H, 6T\} \).
4. \( 3 \cdot 2 \cdot 1 = 6, S = \{ABC, ACB, BAC, BCA, CAB, CBA\} \).
5. There will be three different outcomes on the tree diagram, \( S = \{RB, BB, BR\} \).
6. \( 3 \cdot 2 \cdot 1 = 6 \)
7. \( 5! = 120 \).
8. \( 3,628,800 - 120 = 3,628,680; \ [b] 5! = 120; \ [c] 2,730; \ [d] 306 \).
9. \( 4! = 24 \).
10. \( 20 \cdot 19 \cdot 18 = 6,840 \).
11. Each of the 5 persons will shake hands with 4 other people. \( 5 \cdot 4 = 20 \). However, this includes a second handshake for each pair of people, so we divide this number in half. There are 10.
12. \( 4 \cdot 3 \cdot 2 \cdot 1 = 24 \).
13. \( 1 \cdot 10 \cdot 1 \cdot 10 \cdot 1 \cdot 10 \cdot 10 \cdot 10 \cdot 5 = 50,000,000 \).
14. \( 5 \cdot 4 \cdot 3 = 60 \).
15. \( 2^{10} \cdot 5^{10} = 10^{10} \).

Chapter 3.2, Permutations
1. \( [a] 120, \ [b] 720, \ [c] 5, \ [d] 1 \).
2. \( P(20, 3) = 6,840 \).
3. \( P(7, 4) = 840 \).
4. There are 11 letters of which the M, A, and T are each repeated twice, \( \frac{11!}{2! \cdot 2! \cdot 2!} = 4,989,600 \).
5. \( 7 \cdot 7 \cdot 7 = 7^3 = 2,401 \).
6. Fixing one seat at the table converts the problem to 14 seats (1 vacant) in a row, or 13!
7. ways.
8. \( 10 \cdot 10 \cdot 10 \cdot 10 \cdot 1 \cdot 10 = 10,000 \).
9. Start with 8 ways to place 8 keys on a ring (circular permutations) then multiply by 2 for each key minus 1 (each key could face up or down). Therefore, \( (8 - 1)! \cdot 2^7 = 645,120 \).
10. \( 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 1,000,000 \).
11. \( P(6, 4) = 360 \).
21. \(26 \cdot 25 \cdot 4 \cdot 4 = 10,400.

23. \(4 \cdot 8 \cdot 7 \cdot 3 \cdot 6 \cdot 5 \cdot 2 \cdot 4 \cdot 3 \cdot 1 \cdot 2 \cdot 1 = 967,680.

25. Start with the total number of ways with no restrictions, then subtract the number of ways with all evens, 0 (there is only 2 even elements); and subtract the number of ways for all odds, 3 \(\cdot\) 2 \(\cdot\) 1. That is, 60 \(-\) 0 \(-\) 6 = 54.

27. 8 games are required.

29. The row could begin with a boy or a girl, 2 choices. Then, \(P(5,5)\) ways to seat the boys and \(P(5,5)\) ways to seat the girls. So, there are 28,800 possibilities.

31. \(\frac{15!}{(3!)^5} = 168,168,000.

Chapter 3.3, Combinations

3. \(C(20, 3) = 1,140.

5. \(C(52, 5) = 2,598,960.

7. \(C(3, 1) \cdot C(1, 1) \cdot C(2, 1) = 6.


11. Either 1 B and 2 non-B or 2B and 1 non-B will satisfy, which results in 81.

13. Need to pick 2 kinds, then pick which of the 2 will be 4 of kind, then pick 1 card of the other kind. \(C(13, 2) \cdot C(2, 1) \cdot C(4, 4) \cdot C(4, 1) = 624.

15. \(P(31, 2) = 930.

17. 2\(^5\) = 32

19. \(C(20, 2) \cdot C(15, 2) = 190 \cdot 105 = 19,950.

21. [a] \(C(20, 3) = 1,140.

[b] (2F and 1M) or (3F and 0M): \(C(10, 2) \cdot C(10, 1) + C(10, 3) \cdot C(10, 0) = 45 \cdot 10 + 120 \cdot 1 = 570.

[c] Find the complement to part b, 1,140 \(-\) 570 = 570.

23. [a] Pick a suit, then pick 5 cards in suit, \(C(4, 1) \cdot C(13, 5) = 5,148.

[b] Pick 3 clubs, then pick 2 non-clubs, \(C(13, 3) \cdot C(39, 2) = 211,926.

25. \(C(30, 5) = 142,506.

27. [a] \(C(8, 4) = 70, [b] C(2, 2) \cdot C(8, 2) = 28.

29. To pick the 8 in favor (or 4 against) is \(C(12, 8) = C(12, 4) = 495.

Chapter 3.4, Chapter Review

Mastery Quiz

1. [a], 2. [b], 3. [a], 4. [a], 5. [b], 6. [a], 7. [b], 8. [b], 9. [c], 10. [c]
Review

2. [a] \( P(20, 3) = 6,840 \) [b] \( C(20, 3) = 1,140 \).
3. \( 99 \cdot 26 \cdot 10 \cdot 10 \cdot 10 = 25,740,000 \).
4. \( \frac{40!}{6! \cdot 8! \cdot 8! \cdot 12!} \approx 2.02 \times 10^{24} \).
5. Cannot have 1 M and 3 F because there are only 2 F; thus 2 M and 2 F or 3 M and 1 F,
   \( C(6, 2) \cdot C(2, 2) + C(6, 3) \cdot C(2, 1) = 55 \).
6. First select toppings, then crust, then drink, \( C(12, 3) \cdot 2 \cdot C(6, 1) = 2,640 \).
7. \( 5 \cdot 4 \cdot 9 \cdot 9 \cdot 9 = 131,220 \).
8. \( \frac{11!}{4! \cdot 4! \cdot 2!} = 34,650 \).
9. [a] \( C(12, 3) \cdot C(10, 3) = 26,400 \).
   [b] Use the complement of 0 boys and 6 girls: \( C(22, 6) - C(12, 0) \cdot C(10, 6) = 74,403 \).
   [c] Select 6 from 22 in combination, then select 2 from these 6 in permutation:
   \( C(22, 6) \cdot P(6, 2) = 2,238,390 \).
10. Construct a chart, 5 ways.
11. \( (8 - 1)! = 5,040 \).
12. Row starts with men or with women, \( P(4, 4) \cdot P(4, 4) + P(4, 4) \cdot P(4, 4) = 1,152 \).
13. \( P(10, 4) \cdot P(8, 2) = 282,240 \).
14. \( \frac{n \cdot (n - 1)}{2} = 105 \), where \( n = 15 \).
15. \( C(31, 3) \cdot 4 \cdot 2 = 35,960 \).
16. [a] Number of distinguishable arrangements of 12 people is 12!, with pairing of 2 people
    per room being indistinguishable is \( \frac{12!}{2! \cdot 2! \cdot 2! \cdot 2! \cdot 2!} = 7,484,400 \).
    [b] Pick 3 of the 6 rooms for the men, arrange the 6 men into 3 groups of 2, then arrange
    the 6 women into 3 groups of 2. \( C(6, 3) \cdot \frac{6!}{2! \cdot 2! \cdot 2!} \cdot \frac{6!}{2! \cdot 2! \cdot 2!} = 162,000 \).
17. \( P(2, 1) \cdot P(4, 4) = 48 \).

Chapter 4.1, Basic Concepts

1. \( Pr[7] = \frac{6}{36} = \frac{1}{6} \).
2. \( Pr[H] = \frac{1}{8} \).
3. \( Pr[HHH] = \frac{1}{8} \).
4. \( Pr[T] = 1 - Pr[H] = 1 - \frac{2}{3} = \frac{1}{3} \).
5. \( Pr[8'] = 1 - Pr[8] = 1 - \frac{5}{36} = \frac{31}{36} \).
6. \( Pr[A \cap B] = 0.75 + 0.30 - 0.25 = 0.80 \).
7. \( Pr[A \cup B] = 0.3 + 0.8 - 0.9 = 0.2 \).
8. \( Pr[# \text{will occur at least once in 3 throws}] = 1 - Pr[# \text{will not occur in 3}] = 1 - \left( \frac{5}{6} \right)^3 \approx 0.42 \).
9. \( Pr[\text{at most 3 tails}] = 1 - Pr[4 \text{ tails}] = 1 - \frac{1}{16} = \frac{15}{16} \).
10. [a] \( Pr[\text{at most 3 tails}] = 1 - Pr[4 \text{ tails}] = \frac{15}{16} \).
17. \( \Pr[12 \text{ or } 20] = \frac{6}{36} = \frac{1}{6} \).
19. \( \Pr[A \cup B] = 0.55 + 0.25 - 0 = 0.80. \)
21. \( \Pr[C] = 1 - \Pr[A] - \Pr[B] = 1 - 0.15 - 0.45 = 0.40. \)
23. \( \Pr[\text{at least } 1B \text{ and } 1W] = \frac{n(\text{at least } 1B \text{ and } 1W)}{n(S)} = \frac{C(6,1) \cdot C(4,2) + C(6,2) \cdot C(4,1)}{C(10,3)} = \frac{96}{120} = \frac{4}{5}. \)
25. Pick 1 of 4 suits, then pick 5 cards in that suit,
\( \Pr[\text{same suit}] = \frac{n(\text{same suite})}{n(S)} = \frac{C(4,1) \cdot C(13,5)}{C(52,5)} = \frac{5,148}{2,598,960}. \)
27. Pick 1 of the 4 suits, there are 9 such sequences in each suit,
\( \Pr[\text{straight flush}] = \frac{n(\text{straight})}{n(S)} = \frac{C(4,1) \cdot 9}{C(52,5)} = \frac{36}{2,598,960}. \)
29. \( \Pr[B \text{ before } C \text{ and } D \text{ before } E] = \Pr[B \text{ before } C] \cdot \Pr[D \text{ before } E] = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}. \)

Chapter 4.2, Conditional Probability and Independence

1. [a] \( \frac{1}{3} \), [b] 0.4,
3. \( \Pr[\text{one die is 5 } | \text{ sum is } 8] = \frac{\Pr[\text{one die is 5 } \cap \text{ sum is } 8]}{\Pr[\text{sum is } 8]} = \frac{\frac{2}{36}}{\frac{5}{36}} = \frac{2}{5}. \)
5. \( \Pr[A \cup B] = \Pr[A] + \Pr[B] - \Pr[A \cap B] = \Pr[A] + \Pr[B] - \Pr[A] \cdot \Pr[B] = 0.3 + 0.5 - 0.3 \cdot 0.5 = 0.65. \)
7. [a] 0.21, [b] 0.79, [c] 0.09.
9. [a] 0.8, [b] 0.5, [c] 0.2, [d] 0.26.
11. 0.8.
13. [a] \( \Pr[2G \mid \text{ at least } 1B] = \frac{\Pr[2G \cap 1B]}{\Pr[\text{at least } 1B]} = \frac{\frac{3}{8}}{\frac{4}{8}} = \frac{3}{4}, \) [b] \( \frac{1}{4}. \)
15. Not independent, 0.001, 9 times.
17. 0.42.
19. \( \Pr[1st H \mid \text{ at least } 2H] = \frac{\Pr[1st H \cap \text{ at least } 2H]}{\Pr[\text{at least } 2H]} = \frac{\frac{2}{8}}{\frac{4}{8}} = \frac{3}{4}. \)
21. [a] \( \Pr[2R] = 0.05 + 0.03125 = 0.08125, \) [b] \( \Pr[\text{at least } 1R] = 0.56875. \)
23. \( \Pr[4R \mid \text{ at least } 2R] = \frac{\Pr[4R \cap \text{ at least } 2R]}{\Pr[\text{at least } 2R]} = \frac{\frac{C(10,4)}{C(18,4)}}{\frac{C(10,2) \cdot C(8,2) + C(10,3) \cdot C(8,1) + C(10,4)}{C(18,4)}}. \)
29. 0.4148.
31. [a] 0.20, [b] 0.2667, [c] 0.45.
Chapter 4.3, Bayes’ Theorem

1. \[ \Pr[HIV \mid PR] = \frac{\Pr(HIV \cap PR)}{\Pr(HIV \cap PR) + \Pr(\text{Not HIV} \cap PR)} = \frac{(0.004)(0.98)}{(0.004)(0.98) + (0.996)(0.09)} \approx 0.0419 \]

3. \[ \Pr[HIV \mid NR] = \frac{(0.004)(0.02)}{(0.004)(0.02) + (0.996)(0.91)} \approx 0.0000883. \]

5. [a] \( \Pr[A \mid C] = \frac{0.10}{0.10 + 0.16} = \frac{5}{13} \), [b] \( \Pr[B \mid C] = \frac{0.16}{0.10 + 0.16} = \frac{8}{13} \).

[c] \( \Pr[B \mid D] = \frac{0.24}{0.10 + 0.24} = \frac{12}{17} \), [d] 1.

7. \( \frac{1}{3} \)

9. \( \Pr[A \mid Rotten] = \frac{(0.5) \cdot (0.04)}{(0.5) \cdot (0.04) + (0.5) \cdot (0.06)} = 0.4 \)

11. \( \Pr[\text{Actually true} \mid \text{Identified as lie}] = 0.375 \).

13. \( \Pr[\text{Heads} \mid \text{White}] = \frac{20}{37} \).

15. \( \Pr[\text{Man} \mid \text{Lunch}] = \frac{2}{3} \frac{1}{4} + \frac{1}{3} \frac{1}{2} = 0.5 \).

17. \( \Pr[\text{AAA} \mid \text{Defaulted}] = 0.05 \)

19. \( \frac{2}{3} \).

21. \( \Pr[\text{Two-tail coin} \mid \text{Tails}] = \frac{2}{3} \).

23. \( \Pr[\text{Two-tailed urn} \mid \text{Tails}] = 0.05 \)

25. [a] \( \frac{5}{16} \), [b] \( \frac{1}{2} \), [c] \( \frac{1}{8} \)

27. \( \frac{1}{3} \).

Chapter 4.4, Bernoulli Trials

1. \( 0.2373 \).

3. \( 0.1977 \).

5. \( 0.2254 \).

7. \( 1 - \Pr[\text{success on all 10}] = 1 - b(10; 10) \approx 1 \).

9. [a] \( b(2; 12, 0.05) \approx 0.0988 \), [b] \( 1 - b(0; 12, 0.05) - b(1; 12, 0.05) \approx 0.1184 \).

11. \( b(3; 5, 0.5) + b(4; 5, 0.5) + b(5; 5, 0.5) = 0.5 \).

13. \( 1 - b(0; 6, 0.25) - b(1; 6, 0.25) \approx 0.466.15 \).

17. \( b(2; 4, 0.25) \approx 0.2109 \).

19. Find smallest \( n \).

21. \( 0.1316 \).

23. [a] \( 0.1509 \), [b] \( 0.8814 \).

25. \( \Pr[4 \text{ or } 5] \approx 0.4398 \) is most likely, then \( \Pr[>5] \approx 0.3348 \), and least likely \( \Pr[<4] \approx 0.2254 \).

27. \( 14 \).

29. \( 4 \).
Chapter 4.5, Chapter Review
Mastery Quiz

1. [b], 2. [b], 3. [b], 4. [d], 5. [a], 6. [c], 7. [d], 8. [c] and [e], 9. [c], 10. [b]

Review

1. [a].6, [b].12, [c].2, [d].0.


5. 0.375.

7. 0.6.


11. Pick 2 ranks, then 1 of 2 for triple, pick 3 cards, pick 2 cards,

\[ \Pr[\text{Full House}] = \frac{C(13,2) \cdot C(2,1) \cdot C(4,3) \cdot C(4,2)}{C(52,5)}. \]

13. \( \Pr[\text{5 consecutive cards in same suit}] = \frac{C(4,1) \cdot 10}{C(52,5)}. \)

15. 0.0698.

17. 0.2.

19. 20/21.

21. 0.7.

23. 0.1.

25. \( \Pr[\text{2 M | at least 1 M}] = \frac{C(5,2)}{C(8,2)} \cdot \frac{C(5,1) \cdot C(3,1) + C(5,2) \cdot C(3,0)}{C(8,2)} = \frac{2}{5}. \)

27. \( b(13; 15, 0.5) + b(14; 15, 0.5) + b(15; 15, 0.5) \approx 0.0037. \)

29. 1/120.

Chapter 5.1, Central Tendency

1. 8.

3. 6.

5. 6.44, rounded to the nearest hundredth.


9. 58°

11. $33,305.56.

13. 2.46.

15. 18.46.

17. $0.

19. Group A.

21. Mean 16.7 and median 8.5.

Chapter 5.2, Expected Value and Standard Deviation

1. 1.2.

3. \( E[X] = 3.0, \quad 2.7568. \)

5.

<table>
<thead>
<tr>
<th>( X )</th>
<th>( Pr[X] )</th>
<th>( Product )</th>
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<tbody>
<tr>
<td>25</td>
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<td>5</td>
</tr>
<tr>
<td>10</td>
<td>.4</td>
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<td>.2</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>.2</td>
<td>.2</td>
</tr>
</tbody>
</table>

\( E[X] = 10.2 \)
7. 1.0565
11. \(E[X] = 25, \sigma = 4.33\).
15. 6 people.
19. \(E[X] = 1.5\).
9. 35 cents.
13. 3.25 flips.
17. Expect to lose $0.05 per game.
21. \(E[X] = 1.5, \sigma = 0.91\).

**Chapter 5.3, Normal Random Variable**
1. \[a] 0.5000, \[b] 0.8413, \[c] 0.3413.
3. \[a] 0.5000 \ 0.0618 \ 0.4382, \[b] 0.9987 \ 9772 \ 0.0215,
   \[c] 0.9525 \ 0.0062 \ 0.9463.
5. \[a] 0.8413 \ 0.5000 \ 0.3413, \[b] 0.1587, \[c] 1.0000 \ 0.1587 \ 0.8413.
7. \(0.9082 - 0.0643 \approx 0.8439\).
9. \(1 - 0.0485 \approx 0.9515\).

**Chapter 5.4, Normal Approximation to a Binomial Random Variable**
5. 0.2236.
9. 0.6318.
13. 6, \(-17\%\).
7. 630.
11. 90.7.
15. Approximately 0.

**Chapter 5.5, Chapter Review**

**Mastery Quiz**
1. \[c\], 2. \[b\], 3. \[b\], 4. \[a\], 5. \[b\], 6. \[c\], 7. \[d\], 8. \[c\], 9. \[a\], 10. \[b\]

**Review**
1. \[a\] 28.33, \[b\] 25 and 32, \[c\] 30.
3. 33.33 cents.
5. 2.
7. \[a\] \([0, 1, 2]\), \[b\] \(\left\lfloor \frac{20}{56}, \frac{30}{56}, \frac{6}{56} \right\rfloor\), \[c\] \(\frac{42}{56}\), \[d\] = 0.634.
9. \(a = 0, b = 0.3, c = 0.2, d = 5, e = 1, f = 0.1\).
11. $0.50.
13. 51.
15. \[a\] 40 \[b\] 6.197 \[c\] 0.063 \[d\] 0.0630.
17. 0.0082.
19. 0.0559.
21. Approximately 100\%. 

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Chapter 6.1, Systems of Linear Equations in Two Variables
1. Yes, it is a solution.
3. (3, -2).
5. No solutions, inconsistent, independent.
7. \( \begin{pmatrix} 4 & 14 \\ 3 & 3 \end{pmatrix} \).
9. (0, 0).
11. \( \begin{pmatrix} -1 \\ 3 \end{pmatrix} \).
13. No, it is not a solution.
15. No solution, inconsistent, independent.
17. No solution, inconsistent, independent.
19. (2, 3) consistent and independent.
21. (-3, 2).
23. (1, -2).
25. \( \begin{pmatrix} -25 \\ 2 \\ 11 \\ 2 \end{pmatrix} \).
27. (-3, 0).
29. (-12, -15).

Chapter 6.2, Matrices and Systems of Linear Equations
1. \( 2x - 6y = \frac{1}{4}, -3x + y = 8 \).
3. \( \begin{pmatrix} 63 \\ 29 \\ -114 \\ 29 \end{pmatrix} \).
5. (-1, 2, 3).
7. (-1, 2, 3).
9. No solution, the system is inconsistent.
11. (-3, 2).
13. (7, 3).
15. (2.5, -1).
17. (-2, -1, 4).
19. Infinitely many solutions:
\( x = 3y + 2, y = \text{any number} \).
21. No solution, the system is inconsistent.
23. (4, 0.5, -0.5).
25. No solution, the system is inconsistent.
27. Infinitely many solutions:
\( w = 19 - 2y - 16z, x = -6 + 3y + 4z, y = \text{any number}, z = \text{any number} \).
29. Infinitely many solutions:
\( x = -1 + 4z, y = 4 + 2z, z = \text{any number} \).
31. Infinitely many solutions:
\( v = 6 - 8x + 3y, w = 4 - 4x - 2y, x = \text{any number}, y = \text{any number}, z = -5 \).

Chapter 6.3, Matrix Notation, Algebra, and Inverses
1. A 2x2, B 2x5, C 3x1, D 1x2, E 2x2.
3. \( a_{22} = 9, b_{12} = 12, b_{23} = -1, c_{11} = 2, d_{12} = 3 \).
5. \( I' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, J' = \begin{pmatrix} 3 & 4 & -8 \\ -1 & 9 & 7 \\ 2 & -5 & 6 \end{pmatrix}, K' = \begin{pmatrix} 1 & 3 & 3 \\ 2 & 1 & 3 \\ 2 & 2 & 1 \end{pmatrix} \).
7. \[ X' = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \]
9. \[ \begin{bmatrix} 5a + 1c & 5b + 1d \\ 2a - 3c & 2b - 3d \end{bmatrix} \]

11. \[ A^{-1} = \begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix} \]
13. \[ A^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix} \]

15. [a] E = \begin{bmatrix} 9 & 9 \\ -3 & -3 \end{bmatrix} [b] \begin{bmatrix} 11 & 13 \\ 5 & 3 \end{bmatrix}
17. [a] Not possible. [b] A \begin{bmatrix} -1 \\ -3 \end{bmatrix}

19. [a] \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} [b] \begin{bmatrix} -1 & 3 \\ -2 & 2 \\ -3 & 1 \end{bmatrix} [c] \begin{bmatrix} -1 & 4 \\ 6 & 2 \\ 7 & 0 \end{bmatrix}
21. \[ AB = [-10] \text{ or } -10; \; BA = \begin{bmatrix} -6 & 3 \\ 8 & -4 \end{bmatrix} \]

23. \[ AB = \begin{bmatrix} 7 & -6 & 1 \\ -15 & 12 & 3 \\ -2 & -1 & 8 \end{bmatrix}, \; BA = \begin{bmatrix} 9 & 11 & -10 \\ 3 & 4 & -4 \\ -3 & -1 & 14 \end{bmatrix} \]
25. \[ AB = [35, 19], \; BA \text{ is not possible.} \]

27. \[ \begin{bmatrix} 5 & 7 \\ 6 & 1 \end{bmatrix} = \begin{bmatrix} -4 \\ 3 \end{bmatrix} \]
29. [a] \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} [b] \begin{bmatrix} 2 & -3 \\ -7 & 11 \end{bmatrix} [c] \begin{bmatrix} \frac{3}{8} & \frac{1}{4} & \frac{1}{8} \\ \frac{1}{8} & \frac{3}{4} & \frac{3}{8} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{bmatrix}

31. (-23, 83).
33. \[ \left( \frac{1}{11}, \frac{6}{11} \right) \]
35. \(\left( \text{a} \right) \) \[ \begin{bmatrix} 28 & -19 \\ 11 & -11 \end{bmatrix} \]

Chapter 6.4, The Leontief Input-Output Model

1. [a] A = \[ \begin{bmatrix} 0.25 & 0.6 \\ 0.7 & 0.4 \end{bmatrix} = \begin{bmatrix} \frac{5}{20} & \frac{12}{14} \\ \frac{14}{8} & \frac{11}{2} \end{bmatrix} \], [b] D = [150 70]^t, [c] \[ \frac{12}{3} \begin{bmatrix} 12 \\ 15 \end{bmatrix} \], [d] [4400 5250]^t.

2. [a] A = \[ \begin{bmatrix} 0.2 & 0.5 \\ 0.25 & 0.2 \end{bmatrix} = \begin{bmatrix} \frac{1}{100} & \frac{20}{25} \\ \frac{25}{20} & \frac{16}{15} \end{bmatrix} \], [b] [5 3], [c] \[ \frac{10}{103} \begin{bmatrix} 16 \\ 19 \end{bmatrix} \], [d] D = [1 1]^t.

[e] X = [926.2 1.111 2.609]^t, [f] 5896.9 work-days.

5. [a] A = \[ \begin{bmatrix} 0.2 & 0.1 & 0.5 \\ 0 & 0.1 & 0 \\ 0.2 & 0.2 & 0.3 \end{bmatrix} = \begin{bmatrix} \frac{1}{207} & \frac{63}{18} & \frac{45}{18} \\ 0 & 46 & 0 \\ 18 & 18 & 18 \end{bmatrix} \], [b] [0.4 0.2 0.3], [c] \[ \frac{5}{207} \begin{bmatrix} 63 & 17 \\ 18 & 18 \end{bmatrix} \], [d] D = [1 1]^t.


7. [a] \[ \frac{5}{16} \begin{bmatrix} 5 \\ 2 \\ 4 \end{bmatrix} \], X = [144.4 113.8]^t; x_0 = 122.98.
9. [a] \[ \frac{5}{27} \begin{bmatrix} 7 \\ 1 \\ 2 \end{bmatrix} \], X = [142.2 657.8 1822.2]^t; x_0 = 488.
Chapter 6.5, Chapter Review

1. \((4, 8)\); Consistent; Independent.
3. No solution.
5. \((0, 0, 0)\); Consistent; Independent.
7. $1600 at 10\%$, $3400 at 10.5\%$.
9. \(74.5, 68.5, 82\).
11. \((1, 2)\).
13. \(x = \frac{7}{2}, y = \frac{7}{2}, z = \text{any real number}; (0, 0, 0), (2, -2, 4), (1, -1, 2)\)

15. \[
\begin{bmatrix}
0 & -1 & 6 \\
3 & 1 & -2 \\
-2 & 1 & -2
\end{bmatrix}
\]
17. \[
\begin{bmatrix}
-1 & 1 & 0 \\
-2 & -3 & 2 \\
2 & 0 & -1
\end{bmatrix}
\]
19. Not possible.
21. \[
\begin{bmatrix}
-1 & 1 & 0 \\
0 & -2 & 1 \\
6 & 0 & -3
\end{bmatrix}
\]
23. \[
\begin{bmatrix}
-\frac{1}{2} & 0 \\
\frac{1}{2} & 1 \\
6 & \frac{1}{3}
\end{bmatrix}
\]
25. \[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \frac{1}{9} & \frac{5}{18} & 0 \\
0 & \frac{1}{2} & 2 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]
27. \[
\begin{bmatrix}
5 & -1 & x \\
2 & 3 & y
\end{bmatrix} = \begin{bmatrix} -29 \end{bmatrix}; A^{-1} = \begin{bmatrix} \frac{3}{17} & \frac{1}{17} \\
\frac{2}{17} & \frac{5}{17}
\end{bmatrix}; (-5, 4).
\]
29. $10,000 at 12\%, 12,000 at 13\%, 18,000 at 14.5\%$.

Chapter 7.1, Systems of Linear Inequalities

1. It is not a solution.
3. Draw a dashed line, slope \(-2\), containing \((0, 0)\) and shade the half-plane on right side of line.
5. Draw a solid line, intercepts \((0.5)\) and \((-2, 0)\), and shade the half-plane on right side of line.
7. Draw a vertical line, solid, through \((0, 0)\), and shade the half-plane on right side of line.
9. Corner point: \((-\frac{1}{2}, \frac{3}{2})\).
11. Corner points: \((-\frac{1}{2}, \frac{3}{2}), (2, 4), (4, 4), (1, 0), (4, 0)\).
13. No corner points exist.
15. Corner points: \((0, 0), (0.5), (1.5), (6, 0)\).
17. Corner points: \((0, 0), (0.5), (4.0), (2.3)\).
19. Corner points: \((0, 0), (0.7), (2.6), (5, 2), (6, 0)\).
21. Corner points: \((0.9), (1.6), (6.1), (8.0)\).
23. Corner points: \((0.9), (2.5), (5.1), (8.0)\).
25. Corner points: \((0.9), (1.5), (3.2), (8.0)\).
27. Corner points: \((0.3), (5.1/2), (5.2), (0.2)\).
29. Corner points at \((-3, 0), (1/6, 19/2), (24/7, -2/7), (-3, -7/2)\).
Chapter 7.2, Finding an Optimal Value
1. Max = 10 at \( x = 2 \) and \( y = 2 \), Min = \(-14/3\) at \( x = -4/3 \) and \( y = -2/3 \).
3. Max = 168 at \( x = 0 \) and \( y = 6 \), Min = 0 at \( x = 0 \) and \( y = 0 \).
5. Max = 152 at \( x = 7 \) and \( y = 0 \), Min = 32 at \( x = 0 \) and \( y = 4 \).
7. Max = 11 at \( x = 1 \) and \( y = 5 \).
9. Max = 22 at \( x = 2 \) and \( y = 3 \).
11. Max = 30 at \( x = 2 \) and \( y = 6 \).
13. Min = 22 at \( u = 6 \) and \( v = 1 \).
15. Min = 15 at \( u = 5 \) and \( b = 1 \).
17. Max = 7 at \( x = 5 \) and \( y = -2 \), Min = -3 at \( x = 0 \) and \( y = 3 \).
19. Max = 26/7 at \( x = 24/7 \) and \( y = -2/7 \), Min = -28/3 at \( x = 1/6 \) and \( y = 19/2 \).

Chapter 7.3, Solving Linear Programming Problems Graphically
1. Max = $3350, 25 chairs and 9.5 sofas.
3. Min = $750, 38 of A and 25 of B.
5. Max = $10,312.50, 1875 A and 1875 B.
7. Max = $8,000, 80 corn and 160 oats.
9. Min = $36,92 when \( \frac{1}{11/3} \) sacks of soybean and \( \frac{1}{11/3} \) sacks of oats.
11. Min = $960,000, 24 teachers and 12 aides.

Chapter 7.4, Chapter Review
1. Corner points: \((0,9), (2,5), (5,1), (8,0)\).
3. Corner points: \((0,2), (0,7), (6,2), (1,7)\).
5. Max = 36 at \( x = 10 \) and \( y = 0 \), Min = \(-12\) at \( x = 2 \) and \( y = 0 \).
7. Min = \( \frac{6}{19} \) at \( x = \frac{24}{19} \) and \( y = \frac{98}{19} \).
9. Min = $138,000, 2500 tons A, 6000 tons B.
11. Min = $760 thousand, 31 A and 12 B.

Chapter 8.1, Maximum Problems and Slack Variables
1. \( 20x + 100y + u = 1900, x + 50y + v = 500, 2x + 20y + w = 240 \).
3a. \( x + y + u = 6, y + v = 5 \).
3b. \(-x + 2y + f = 0\).
3c. 

<table>
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<th>( x )</th>
<th>( y )</th>
<th>( u )</th>
<th>( v )</th>
<th>( f )</th>
<th>( 1 )</th>
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</tr>
<tr>
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<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>(-1)</td>
<td>(-2)</td>
<td>0</td>
<td>0</td>
<td>1</td>
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</tbody>
</table>
3d. \( 2 \)
3e. \( 2 \)

3x + 2y + u = 12
Appendices

5a. \( x + y + v = 5 \).
5b. \(-5x - 4y + f = 0\).

\( 4x + 5y + w = 13 \)

5c.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( u )</th>
<th>( v )</th>
<th>( w )</th>
<th>( f )</th>
<th>( 1 )</th>
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</tr>
<tr>
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<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>13</td>
</tr>
<tr>
<td>-5</td>
<td>-4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
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</tbody>
</table>

5d. 2
5e. 3

3x - 2y + z + u = 8.

7a. \(-4x + 3y + 2z + v = 4\).
7b. \(2y - 5z + f = 0\).

3x + y - 6z + w = 6.

7c.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( z )</th>
<th>( u )</th>
<th>( v )</th>
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<tr>
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<td>2</td>
<td>-5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

7d. 3
7e. 3

Chapter 8.2, The Simplex Method

1. Max \( f = 11, x = 1, y = 5 \).
3. Max \( f = 30, x = 2, y = 6 \).
5. Max \( f = 90, x = 30, y = 0 \).
7. Max \( f = 79, x = 9, y = 4 \).
9. Max \( f = \frac{75}{4}, x = 0, y = \frac{9}{4}, z = \frac{19}{4} \).

11. Max profit \$90, 0 A, 0 B, and 3 C.

13. Max income \$20,220, 16 inexpensive, 30 expensive, and 14 medium.

15. Max income \$82 million, \$0 home, \$600 million car, and \$200 million securities.

Chapter 8.3, Duality and Minimum Problems

1. Min \( y_0 = 1900y_1 + 500y_2 + 240y_2 \).
3. No solution.

\[ 20y_1 + y_2 + 2y_3 \geq 20, \]
\[ 100y_1 + 50y_2 + 20y_3 \geq 300, \]
\[ y_1, y_2, y_3 \geq 0. \]

5. Primal solution: \( y_1 = 5, y_2 = 7, x_1 = x_2 = 0, x_3 = 1, y_0 = 53 \).
7. Primal solution: \( y_1 = \frac{8}{11}, y_2 = \frac{2}{11}, y_3 = x_1 = x_2 = 0, x_3 = \frac{13}{11}, y_0 = \frac{82}{11} \).

9. Minimal cost $36 \frac{12}{13}$ using \( 1 \frac{11}{13} \) sacks of soybeans and \( 1 \frac{11}{13} \) sacks of oats.

11. 4 days A and 2 days B.

Chapter 8.4, Mixed-Constraint Linear Programs
No problems in this section.

Chapter 8.5, Chapter Review
1. Max \( f = 43, x = 2, y = 7 \).
3. Max \( f = 59, x = 11, y = 5 \).
5. \( x_1 = 0, x_2 = 5, x_3 = 1, y_1 = 2, y_2 = 0, y_0 = 10 \).
7. \( y_1 = \frac{62}{7}, y_2 = \frac{60}{7}, x_1 = 0, x_2 = \frac{10}{7}, x_3 = 0, y_0 = \frac{428}{7} \).
9. Min \( T = 18, x = 2, y = 4 \).
11. Max $82,500, 125 X, 50 Y,$ and 0 Z.

Chapter 9.1, Transition Matrices
1. \[
\begin{bmatrix}
0.6 & 0.4 \\
0.3 & 0.7
\end{bmatrix}
\]
3. 44.4%.
5. 42.9% Taylor's; 57.1% Sower's.
7. Is a transition matrix.
9. Not a transition matrix, the sum of the elements in last row is not 1.
11. \([a] \begin{bmatrix} 0.48 & 0.52 \end{bmatrix}, [b] \begin{bmatrix} 0.392 & 0.608 \end{bmatrix}\).
13. \[
\begin{bmatrix}
0.5 & 0.5 \\
0.4 & 0.6
\end{bmatrix}
\]
55%.
15. \[
\begin{bmatrix}
0.5 & 0.2 & 0.3 \\
0.4 & 0.4 & 0.2 \\
0.3 & 0.1 & 0.6
\end{bmatrix}
\]
Approximately 40%.
17. 64%.
19. 25%.
21. 49.6%.
23. \( P_2 = [0.36 \ 0.16 \ 0.48]; P_4 = [0.48 \ 0.25 \ 0.27] \).
25. \[
\begin{bmatrix}
16 & 27 & 47 \\
90 & 90 & 90
\end{bmatrix}
\]
\[
\begin{bmatrix}
158 & 141 & 376 \\
675 & 675 & 675
\end{bmatrix}
\]
20%. 27. 8.5%.
29. No.
Chapter 9.2, Regular Markov Chains
1. Yes. 3. No.
5. No. 7. No.
9. Yes. 11. \[
\begin{bmatrix}
20 \\ 14 \\ 13 \\
47 \\ 47 \\ 47
\end{bmatrix}
\]
15. \[
\begin{bmatrix}
4 \\ 3 \\
7 \\ 7
\end{bmatrix}
\]
17. [a] Yes \[
\begin{bmatrix}
\frac{1}{3} \\ \frac{2}{3}
\end{bmatrix}
\], [b] No, [c] No.
19. [a] Yes \[
\begin{bmatrix}
\frac{3}{11} \\ \frac{5}{11} \\ \frac{3}{11}
\end{bmatrix}
\], [b] No, [c] Yes \[
\begin{bmatrix}
\frac{1}{8} \\ \frac{3}{8} \\ \frac{3}{8} \\
\frac{8}{8} \\ \frac{8}{8} \\ \frac{8}{8}
\end{bmatrix}
\]
21. 44%.
23. \[
\begin{bmatrix}
30 \\ 21 \\ 16 \\
67 \\ 67 \\ 67
\end{bmatrix}
\]
25. 58%.
27. \[
\frac{43}{91}
\]

Chapter 9.3, Absorbing Markov Chains
3. Yes.
5. No.
7. \[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0.4 & 0.6 & 0
\end{bmatrix}
\]
9. [a] 0, [b] approximately 18.
11. [a] States 1 and 2; Yes, [b] None; No.
13. [a] State 3; Yes, [b] States 2 and 4; Yes. 15. 47%.
17. 6%; 99%.
19. 67%.
21. 4 times.
23. 60%.

Chapter 9.4, Chapter Review
1. None.
3. None.
5. Irreducible.
7. \[
\begin{bmatrix}
0.25 & 0.50 & 0.25
\end{bmatrix}
\]
9. \[
\begin{bmatrix}
0 & 1 & 0 \\
0 & 1 & 0 \\
0 & 1 & 0
\end{bmatrix}
\]
11. [a] \[
\begin{bmatrix}
W \\
D
\end{bmatrix}
\]
\[
\begin{bmatrix}
0.6 & 0.4 \\
0.13 & 0.87
\end{bmatrix}
\]
, [b] ~78%, [c] ~75%.
13. [a] \[
\begin{bmatrix}
1 \\ 2 \\
3
\end{bmatrix}
\]
[b] ~30%, [c] 0.
15. 2.
Chapter 10.1, Game Trees, Pure Strategies, and Matrix Games

1. I: Bluff, Not Bluff; II: Call, Not Call,
   I Bluff\(\begin{bmatrix} (-5,5) \\ (1, -1) \end{bmatrix}\),
   I Not\(\begin{bmatrix} (5, -5) \\ (0, 0) \end{bmatrix}\).

3. Let \(M = \text{in the mall} \) and \(N = \text{near the mall} \). Both banks: \(\{M, N\} \).
   \[
   \begin{pmatrix}
   M & N \\
   0.58 & 0.52 \\
   0.65 & 0.45
   \end{pmatrix}
   \]

5. Strategies (AA, A) and (AN, A) correspond to the bath AA.
   Strategies (NN, N) and (NA, N) correspond to the path ANA.
   Strategy (AA, N) corresponds to the path ANA.
   Strategy (AN, N) corresponds to the path ANN.
   Strategy (NA, A) corresponds to the path NAA.
   Strategy (NN, A) corresponds to the path NAN.

7. \([b]\) both players: \(\{1, 2\} \), \([c]\) \[
   \begin{bmatrix}
   1 & 2 \\
   1 & 5 \\
   2 & -5
   \end{bmatrix}
   \]

9. \([b]\) both players: \(\{S, D\} \), \([c]\) \[
   \begin{bmatrix}
   S & D \\
   (10, 10) & (10, 100, 000) \\
   (1, 000, 000) & (-1, -1)
   \end{bmatrix}
   \]

11. \([b]\) both stations: \{Switch, Not switch\}, \([c]\) \[
   \begin{pmatrix}
   S & N \\
   0.70 & 0.65 \\
   0.60 & 0.40
   \end{pmatrix}
   \]

Chapter 10.2, Solving Pure-Strategy Matrix Games

1. \((\alpha_1, \beta_2); v = 4\).
3. \((\alpha_3, \beta_2); v = 5\).
5. \((C, C); v = (-10, -10)\).
7. \((\alpha_1, \beta_1); v = 3\).
9. \((\alpha_3, \beta_3); v = 5\).
11. \((\alpha_2, \beta_2); v = 4\).
13. \((\alpha_1, \beta_3) \text{ or } (\alpha_3, \beta_3); v = 3\).
15. Roy should campaign 2 days in northern district and Cal 2 in southern, Roy will get 55%.
17. Station II, no change; station I, lower; 1% goes to station I.
19. \((S, D) \text{ and } (D, S) \text{ are the equilibrium pairs.}\)

Chapter 10.3, Solving Mixed-Strategy Games

1. Evil Scientist should go to Paris \(\frac{1}{3}\) of the time and to Athens \(\frac{2}{3}\) of the time.

James Bond should go to Paris \(\frac{2}{3}\) of the time and to Athens \(\frac{1}{3}\) of the time.

The value of the game is \(-\frac{8}{3}\).
3. The investor should put $5,000 into stocks and $25,000 into bonds.
5. \(x_1 = \frac{1}{2}, x_2 = \frac{1}{2}, y_1 = \frac{1}{4}, y_2 = \frac{3}{4}, v = \frac{5}{2}\).

7. \(x_1 = 1, x_2 = 0, y_1 = 1, y_2 = 0, v = 3\).

9. \(x_1 = \frac{13}{51}, x_2 = \frac{38}{51}, y_1 = \frac{44}{51}, y_2 = \frac{71}{51}, v = \frac{397}{510}\).

11. \(x_1 = 0, x_2 = \frac{6}{7}, x_3 = \frac{1}{7}, y_1 = \frac{6}{47}, y_2 = 0, y_3 = \frac{1}{7}, y_4 = 0, v = \frac{20}{7}\).

13. \(x_1 = \frac{1}{2}, x_2 = 0, x_3 = \frac{1}{2}, x_4 = 0, y_1 = 0, y_2 = \frac{5}{8}, y_3 = 0, y_4 = \frac{3}{8}, v = \frac{5}{2}\).

15. \(x_1 = \frac{3}{4}, x_2 = \frac{1}{4}, x_3 = 0, x_4 = 0, y_1 = \frac{27}{28}, y_2 = \frac{1}{28}, y_3 = 0, y_4 = 0, v = \frac{19}{40}\).

17. \(x_1 = \frac{3}{7}, x_2 = \frac{1}{4}, x_3 = \frac{4}{7}, x_4 = \frac{1}{4}, y_1 = \frac{6}{7}, y_2 = 0, y_3 = 0, y_4 = \frac{11}{7}, v = \frac{11}{7}\).

19. Both players show one finger \(\frac{1}{2}\) of the time; \(v = 0\).

21. The plant should dump 90\% of the time in the country and 10\% of the time in the stream.

The inspector should go to the country 90\% of the time and to the stream 10\% of the time; \(v = \epsilon 70\).

23. The store should use 75\% mail and 25\% door to door.

The citizens should use 50\% mail and 50\% door to door.

No, they will only collect 250 signatures.

25. Not drink; expected time of survival is 10 hours.

27. Challenger: prepare for I \(\frac{7}{20}\), II \(\frac{13}{20}\); Champion: serve I \(\frac{1}{4}\), II \(\frac{3}{4}\).

29. \$500.

Chapter 10.4, Chapter Review

1. Both players' strategies are \{N, E\}, the payoff matrix is

\[
\begin{pmatrix}
N & E \\
\begin{pmatrix}
200,000 & 300,000 \\
75,000 & 50,000 \\
\end{pmatrix}
\end{pmatrix}
\]

3. \(X = \) purchase 100 g, \(Y = \) purchase 200 g, \(B = \) busy week, and \(S = \) slow week.

\[
\begin{array}{c|cc}
& B & S \\
\hline
\text{Nature} & 30 & 50 \\
\text{Store} & 100 & -10 \\
\end{array}
\]

5. \((\alpha_X, \beta_Y); v = 3\) \(x_1 = \frac{4}{5}, x_2 = \frac{1}{5}, y_1 = \frac{2}{5}, y_2 = \frac{3}{5}, v = \frac{18}{5}\)

7. \(x_1 = \frac{4}{5}, x_2 = \frac{1}{5}, y_1 = \frac{2}{5}, y_2 = \frac{3}{5}, v = \frac{18}{5}\)

9. \((\alpha_X, \beta_Y)\) or \((\alpha_Y, \beta_X)\) or \((\alpha_Y, \beta_X)\) or \((\alpha_Y, \beta_X)\); \(v = 1\)

11. \((\alpha_X, \beta_Y); v = 2\) \(x_1 = \frac{4}{5}, x_2 = \frac{1}{5}, y_1 = \frac{2}{5}, y_2 = \frac{3}{5}, v = \frac{18}{5}\)


15. Buy 100 gallons 85\% of the time.

17. 0.3 paintings, 0.7 books.

19. I: Park; II: Main Street; 0: Yes.

21. \$3333.33.