FINAL EXAM
SPRING 2011
MATH 165
1. (4 points) Show that \( \lim_{x \to 3}(2x + 1) = 7 \), using the \( \epsilon-\delta \) definition of limit.
2. (12 points) If the limit exists, find its value. If it does not exist explain WHY.

(a) \( \lim \limits_{x \to -1} \frac{x^2 - 1}{x^2 - x - 2} \)

(b) \( \lim \limits_{x \to 1} \frac{\sqrt{5x^2 + 4}}{x^2 + 2} \)

(c) \( \lim \limits_{x \to 2^-} \frac{\sqrt{x - 2}}{x^2 + 1} \)

(d) \( \lim \limits_{x \to \infty} \frac{-1 + 2x + x^3}{4 - x^2 - 3x^3} \)
3. (12 points) Compute the derivatives of the following functions:

(a) \( f(x) = \frac{2}{\sqrt{x}} + \tan(5x). \)

(b) \( g(x) = \cos(\sin x^2). \)

(c) \( h(x) = \int_1^{x^2} \tan t \, dt. \)
4. (6 points) Do implicit differentiation to find the slope of the tangent line to the curve 
\[ y^2 - x + 1 = 0 \]
at the point \((2, -1)\).
5. (12 points) Given \( f(x) = x^3 - 3x + 2 \),

(a) Where is \( f \) increasing?

(b) Where is \( f \) concave up?

(c) What are the local maxima and minima?
6. (6 points) Find the horizontal and vertical asymptotes of the curve

\[ y = \frac{3x^2 + 2x + 1}{x - 1}. \]

Justify your answers with the appropriate limits.
7. (6 points) Find the point on the curve $x^2 - y^2 = 1$ that is closest to $(2, 1)$. 
8. (4 points) Set up (but do not compute) the Riemann Sum for the area under the graph of the function \( f(x) = x^3 - x \), with \( 2 \leq x \leq 3 \).

Use RIGHT-HAND endpoints and 5 subintervals.
9. (16 points) Evaluate the following integrals:

(a) \( \int \cos x \sin^5 x \, dx \).

(b) \( \int x \sec(x^2) \tan(x^2) \, dx \).

(c) \( \int_0^1 \frac{x}{\sqrt{3x^2 + 1}} \, dx \).

(d) \( \int_{-1}^1 \frac{x}{\sqrt{x^4 + 5}} \, dx \).
10. (6 points) Sketch the region enclosed by the curves

\[ x = y, \quad x = y^3. \]

Find the area of the region.
11. (8 points) Let \( A \) be the region bounded by the graphs of 
\[ y = \cos x \ , \quad y = \sin x \ , \quad \text{the } y\text{-axis}. \]

Set up (but do not compute) the integrals to find the volume of the solid generated by revolving \( A \) about

(a) the \( x\)-axis.

(b) the line \( x = -1 \).
12. (6 points) A force of 35N is required to hold a spring stretched 10 cm from its natural length. How much work is done when stretching the spring 5 cm further?
Bonus (4 points): Evaluate
\[ \lim_{x \to 1} \frac{\sqrt{x} - 1}{\sqrt{x} - 1}. \]

Bonus (4 points): Evaluate
\[ \int \frac{dx}{(2 + \sqrt{x})^3}. \]